K21U 1833

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree CBCSS (OBE) Reg./Sup./Imp. Examination, November 2021
(2019 – 2020 Admission)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
3C03 MAT-PH : Mathematics for Physics – III

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Calculate $\iint_R \frac{\sin x}{x} dA$ where R is the triangle in the XY-plane bounded by X-axis, the line $y = x$ and the line $x = \frac{\pi}{2}$.
2. Find a vector parallel to the line of intersection of the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.
3. The Laplace transform of t^2 is
4. What is fundamental period for a periodic function ?
5. Evaluate $\int_0^1 \int_0^{1-x} \int_{x+z}^1 dydzdx$.

PART – B

Answer **any seven** questions. **Each** question carries **two** marks.

6. Find the distance from (1, 1, 3) to the plane $3x + 2y + 6z = 6$.
7. Find an equation for the tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at the point (–2, 1).
8. What are the directions of zero change in $f(x, y) = \frac{1}{2}(x^2 + y^2)$ at (1, 1) ?
9. If r is a differentiable vector function of t with constant length prove $r.\frac{dr}{dt} = 0$.

P.T.O.

10. Find the average value of $F(x, y, z) = xyz$ throughout the cubical region D bounded by the coordinate planes and the planes $x = 2$, $y = 2$, $z = 2$ in the first octant.

11. Find the Laplace transform of $e^{-t}\sin^2 t$.

12. If $H(s) = \frac{1}{(s^2 + \omega^2)^2}$ find h(t).

13. Prove that the Fourier series of an odd function f(x) of period 2L is a Fourier Sine Series.

14. What is the orthogonality relations of the trigonometric system ?

15. Find the Principal Unit Normal Vector for the curve $r(t) = \cos 2t\, i + \sin 2t\, j$.

16. Find $\mathcal{L}(f(t))$ where $f(t) = \begin{cases} \cos\left(t - \frac{\pi}{3}\right) & \text{if } t > \frac{\pi}{3} \\ 0 & \text{otherwise} \end{cases}$.

PART – C

Answer **any four** questions. **Each** question carries **three** marks.

17. Sketch the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$ and find area of this region.

18. Find the curvature of the circular helix $r(t) = a\cos t\, i + a\sin t\, j + bt\, k : a, b \geq 0$, $a^2 + b^2 \neq 0$.

19. Solve $y' - y = t$, $y'(0) = 1$, $y(0) = 1$ using Laplace mehtod.

20. Using convolution solve $y'' + 3y' + 2y = r(t)$: with $y'(0) = y(0) = 0$ and $r(t) = \begin{cases} 1 & \text{if } 1 < t < 2 \\ 0 & \text{otherwise} \end{cases}$.

21. Verify Fubini's theorem for $f(x, y) = 100 - 6x^2y$ for $0 \leq x \leq 2$ and $-1 \leq y \leq 1$.

22. Let f(x) be a function of period 2π such that $f(x) = \begin{cases} x, & 0 < x < \pi \\ \pi, & \pi < x < 2\pi \end{cases}$. Find the Fourier series for f(x) in this interval.

23. Find the volume of the solid region bounded above by the Paraboloid $z = 9 - x^2 - y^2$ and below by the unit circle in the xy-plane.

PART – D

Answer **any two** questions. **Each** question carries **5** marks.

24. Evaluate $\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2} dxdy$ by applying the transformation $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$.

25. Solve the Laguerre's differential equation $ty'' + (1-t)y' + ny = 0$ to identify the Laguerre polynomials. Also prove these polynomials are defined by Rodrigue's formula.

26. A sinusoidal voltage $E = \sin\omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function with period $\frac{2\pi}{\omega}$ and is given by $u(t) = \begin{cases} 0 & -L < t < 0 \\ E\sin\omega t & 0 < t < L \end{cases}$.

27. Find each of the directional derivatives.

    a) $D_u f(2, 0)$ where $f(x, y) = xe^{xy} + y$ and u is the unit vector in the direction of $\theta = \frac{2\pi}{3}$.

    b) $D_u f(x, y, z)$ where $g(x, y, z) = x^2z + y^3z^2 - xyz$ in the direction of $v = (-1, 0, 3)$.

    c) In what direction does f and g change most rapidly and what are the rate of change in these directions.
